METHODOLOGY ON THE DANUBE'S BANK.
Amphorae and Bags at Capidava

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THE PROBLEM

Recently came true an expected book: the pottery collection from the so-called *Guard House* from Capidava fortress (*Opriş 2003*; printed 2004). The site is one of the most assiduous archaeological prospected on the Lower Danube, down between Wars. The collection is almost sure from the end of the third quarter of the sixth century (*Opriş 2003*, p. 33) and set out the most impressive assemblage of entire shapes of all kind. For example, there are no less than 10 hand-made vessels (or slow wheel made) and other large sherds (*Opriş 2003*, p. 102–110, cat. 250–266), that is the most relevant group of artifacts of its kind from all Danube frontier.

Of course, the amphorae are very well represented too, in a military site. About some of them we will have a debate.

Years ago, *before* his dissertation (2000), Ioan Carol Opriş asked me to help him to get capacity figures for amphorae from Capidava. What I did. In the recent book he is publishing a table (nr. 2, p. 46–48), comparing the provided results, under the label "mathematical estimation", with what he is calling "experimental measurement". The author is somehow mysterious about the "experiment", telling us just something about filling the recipients with water (p. 45). Regarding what I am going to debate, I have to uncover the technological tip: he used plastic bags, soft and thin, to protect the pottery from water, avoiding so the absorption or leaking. I had to waste the "secret" for assure the reader that the author took some precautions. From the table of contents (graphs list; page 49 as well) I found out also that Dr. Opriş remembered even the rule of filling up the recipients to the neck's diameter. Apparently, the trouble we are going trough is not due to negligence.

Let's see the table (table 1, below):

Columns signification:

1. inventory number (Capidava repository)
2. number in the catalogue (2003)

EPHEMERIS NAPOCENSIS, XIII, 2003, p. 277–294

3. type of the amphora, conforming to the author
4. result of the "mathematical estimation" (1999); litres
5. exclamation mark (pointing out "significant variance")
6. result of the "experiment", litres

Table 1

The Problem

1. inv.	2. cat.	3. type	4. math.	5. !	6. bag
4488	1	Spatheion	1.37	No	1.325
5054	3	Spatheion	2.13	Yes	1.725
4781	11	LR 1	19.67	Yes	21.050
3140	15	LR 1	15.08	No	15.350
7701	102	LR 3 (var)	1.47	No	1.750
5169	113	Zeest 99	2.97	No	3.100
4479	115	Zeest 99	6.06	No	5.450
4901	127	Antonova V	3.00	Yes	2.400
5005	128	Antonova V	2.08	No	2.100
4484	130	Antonova V	2.40	No	2.500
4649	134	Antonova V	2.15	No	2.100
1639	146	Antonova V	2.21	No	2.150
5044	165	Opaiţ B V	1.51	No	1.500
4486	167	Opaiţ B V	1.64	No	1.850
3142	169	Antonova IX	16.12	Yes	15.000
4622	175	de masă tip 1	9.02	Yes	9.500
5201	184	de masă tip 2	12.75	Yes	13.500

The full list has 71 positions, i.e. complete amphorae. Here we have only the positions with both, mathematical and "experimental" data.

The disparity of data is great enough to push us, together, out of science. What is to note, yet, is that the author seems non-perceptive when comes with proportions. The marked items are around half litre disagree or more, no matter size. He is pointing out items cat. nos. 3 and 127 (errors from 23.5 to 25%), but also cat. 11, 175 and 184 (5–6.5%), slipping over cat. 115 and 167 (over 11%) and 102 (16%! may I?).

There are also some decent results, like cat. 128, 165 (under 1%), or cat. 15, 134, 146 (under 3%). In the situation given, nevertheless, the proper result looks more like a lucky chance. To feel better, let's say that the averages of the two series of data are eccentric only with 1.232%.

About why the "mathematical estimation" is accredited with two decimals, and the bag experiment with three – better not talk.

Of course, the situation given is anxious. How much water could somebody waste pouring it from a recipient to another? How wrong can be a formula long time tested? As I already mentioned years ago (*Teodor 1998*, p. 23), the formula implemented in the database was successful tested against AutoCAD measurements, given errors up to 4%. True, the previous tests were made on pots from barbarian age, not on amphorae, but the theoretical source for such disastrous errors was far from obvious, because the difference between a pot and a typical amphora shape is less than supposed, namely the median quasi-cylindrical segment (see Appendix, at the end of the paper, and fig. 6). A debate about pottery capacity could look scary for an archaeologist, but we are talking here about adding together segments considered as truncated cons, and the formula for the capacity of the truncated con is a toy for a 12 years kid, and some corrections for vaulting walls of the recipient. I suspected this last factor as being the source of errors, because formula used years ago was pretty intuitive. I am using for some time a new formula for arch correction, based on a solid geometrical concept, *spherical sector* ($2/3 * Pi * r^2 h$; see *Memorator 1958*, 174; see Appendix for mathematical signs). The new formula provided results generally a bit greater, but was not improving the comparison with the *bags experiment*. I tried then something new, replacing the calculations based on truncated con (corrections added) with truncated paraboloid, solution described on detail in Appendix, but, again, there was no cure for the studied disease.

Climbing through my database, I noticed a thing: scales were changing all the time; therefore I took a closer look:

Columns meaning for Table 2:

1. cat =	number in catalogue 2003
2. ct_H =	height in catalogue (cms), except cat. 102 (diameter)
3. Hp_ct =	difference between columns 2 and 4
4. I_pl =	height of the item as a produce of drawing height and displayed scale
5. sc99 =	scale used in 1999 drawings (used in capacity calculation with Compass database); only denominator of the fraction (on the first raw, for "3.63", read "1:3.63")
6. 99_03 =	report between columns 5 and 7
7. sc03 =	scale of the drawing in 2003

Note that the table is ordered for column 2.

Studying the table 2, we notice that it is not much of stringency. The amphorae between 30 and 35 cms. height are presented in 1999 stuff to scales ranged between 1:3.28 and 1:3.95, that should mean an intention around 1:3.5. The larger artifacts,

Table 2

Troubles with scales

1. cat.	2. ct_H	3. Hp_ct	4. L_pl	5. sc99	6. 99_03	7. sc03	comment
102	13.6	6.25%	12.75	3.630	48.40%	7.50	broken lip; columns 2, 4 = maximum diameter
134	29.0	-2.23%	29.65	3.550	50.41%	7.04	
146	30.2			3.260			missing drawing 2003
130	31.2	-8.88%	33.97	3.280	44.61%	7.35	
128	31.5	-6.05%	33.41	3.436	47.42%	7.25	
175	32.6	-5.02%	34.24	3.409	38.64%	8.82	
127	33.0			3.950			missing drawing 2003
165	35.0	0.17%	34.94	3.861	65.64%	5.88	
167	35.4	5.52%	33.44	3.846	69.23%	5.56	
3	40.0	0.56%	39.78	5.760	102.53%	5.62	
113	42.0	-0.53%	42.22	5.700	71.82%	7.94	
1	43.8	2.78%	42.58	2.381	42.38%	5.62	
169	44.0	-4.01%	45.76	4.734	55.86%	8.47	
115	44.5	6.37%	41.67	5.747	75.86%	7.58	
184	47.0	-2.16%	48.02	5.454	63.27%	8.62	
15	50.7			6.430			missing drawing 2003
11	55.2	0.06%	55.17	7.290	87.48%	8.33	

from 40 to 55 cms. height, are, for a change, as different as can be, ranging from 1:2.38 to 1:7.29. In 2003 published drawings can't be observed any rule, scale displayed ranging random between 1:5.62 to 1:8.82. For sure, numbers and scales make Dr. Opriş uncomfortable. I can understand, therefore, his bent for practical works, as filling jugs with water.

Armed with this tiny certitude and lots of patience, I took it over, as a brand new start, with parallel measurements in AutoCAD and Compass Database, laying on drawings last published (assumption: better). For the both I considered as artifact height that published in catalogue, not that resulted from graphic scale, as a consequence of the fact that using a measuring tape is simpler as computer applications, therefore the chance for error is lower. For the procedure – see the next section of the paper.

The results are displayed in Table 3 (below), for which:

1. no. cat = catalogue number
2. Cmps parabol. = Compass measurement (using the new paraboloid formula), following the graphic scale

3. comp. 4 to 2 = report between columns 4 and 2 (percent)
4. 2 + corr.scale = same as column 2, with the scale from catalogue
5. comp. 6 to 4 = report between columns 6 and 4 (percent)
6. capacit. ACAD = AutoCAD measurement, following the procedure described below, in the next section
7. comp. 6 to 8 = report between columns 6 and 8 (percent)
8. bag experiment = figures reported by I. Opriş

Table 3

Compared capacities

1. no. cat.	2. Cmps. parabol.	3. comp. 4 to 2	4. 2 + corr. scale	5. comp. 6 to 4	6. capacit. ACAD	7. comp. 6 to 8	8. bag experim.
1	1.48	8.67%	1.62	-1.35%	1.60	17.03%	1.33
3	1.89	8.13%	2.06	-3.19%	2.00	13.71%	1.73
11	19.79	-3.29%	19.16	3.51%	19.86	-6.00%	21.05
102	1.40	18.46%	1.72	2.89%	1.77	1.30%	1.75
113	3.07	4.53%	3.22	7.07%	3.46	10.46%	3.10
115	5.04	21.26%	6.40	-4.80%	6.10	10.71%	5.45
128	2.41	-12.64%	2.14	1.94%	2.18	3.85%	2.10
130	2.52	11.35%	2.84	0.32%	2.85	12.37%	2.50
134	2.39	-12.97%	2.12	-4.09%	2.03	-3.35%	2.10
165	1.71	1.41%	1.73	1.44%	1.76	14.68%	1.50
167	1.82	4.47%	1.90	2.57%	1.95	5.23%	1.85
169	16.88	-19.97%	14.07	2.24%	14.39	-4.23%	15.00
175	9.71	-10.19%	8.82	2.33%	9.03	-5.25%	9.50
184	12.76	-12.55%	11.33	5.09%	11.94	-13.06%	13.50

As I am going to argue below, the ACAD procedure is far to be safe. In this context, yet, ACAD is in a referee position, and the main comparison terms are related to calculations made by this universal tool. The Compass figures are balancing between –4.8 and 5.09%, which is far from excellent, but fair. The bags experiment goes further, in the limits –13.06 and 17.03%, which is quite loose. For instance, for an amphora that have exactly 3.275 l. (i.e. one *congius*), and error of +17% means a result of 3.832 l., which is closer by half *modius* (4.366 l.); this way, we get a recipient for *solids*, not for *liquids*. What could be worst for historical rendition?

As about the third column – it is close to disaster, with figures between minus 19.97 and plus 21.26%. What is instructive here is that in compared columns were used *the same measurement figures, except the scale*. For errors of scale looking "human" (we are all people and are making little mistakes), like –8.88 to 6.37%, we get capacity errors on the triple range. Why so? Because to monster named *Pi*.

Data from the third table is processed in statistic terms, in table 4 (below). The converse error between catalogue heights and scales & drawings is 3.6%, in average, but almost nothing as the average of errors sum (0.5%). That's statistic: perverse... The errors behave random, by nature. One is up, next is down. The errors have a trend that points the true.

On the second row of the table 4 we see, again, how the little errors in scale or drawing grow huge and give birth for aberrant creatures (see column 4): average error of 3.61 generates capacity average error of 10.71, which is three times. If we multiply 3.61 with 3.141592653 (*Pi*), should be 11.34115. The difference (0.63115) represents the measurement error on reading the ruler, on paper; judged else (0.63115/ 11.34115/ Pi), we have 0.0177, respectively 1.77%; for the drawing corresponding to catalogue number 1 (*Opriş 2003*, plate XVII), 1.77% means a reading error of 0.134 mm. for the amphora height, and 0.03 mm. (!) reading error for maximum diameter. That is quite the human eye limit.

On the last column of the second row we find a confirmation that errors trend to zero.

Table 4

Average and global errors

No.	First term	Second term	aver. error (%)	glob. error (%)
1	Heights according to catalogue	Heights according to graphic scale	3.61	0.51
2	Compass capacities according to catalogue	Compass capacities according to graphic scale	10.71	0.48
3	ACAD measurements	Compass capacities according to height in catalogue	3.06	1.14
4	ACAD measurements	"experiment"	8.66	4.10
5	Compass capacities 1999 drawings and measurement	"experiment"	7.75	1.23
6	Compass volume according to height in catalogue	"experiment"	9.84	4.12

note that:

– aver. error = average error (no matter positive or negative);

– glob. error = global error (the average of errors' sum; like: ((10)+(–9))/2=0.5)

What should be the meaning of that 1.14% global error on line 3? Obviously, Compass System gives 1% less than ACAD, or conversely, the procedure followed for ACAD gives something too much (it is not quite clear for me, just now). The average error is also acceptable (3%), within the condition that *any* error is greater than 7%.

The last three lines spare too much talk. There is yet a surprise. The better figures came from comparison with drawings from 1999 (line 5), not with those "improved" from 2003. Seeing the figures I recollected what a younger colleague of mine (a becoming ceramist) said: "OK. That's that! You have to measure them again. But what will you do *next time*?". Frankly – I don't have a clue...

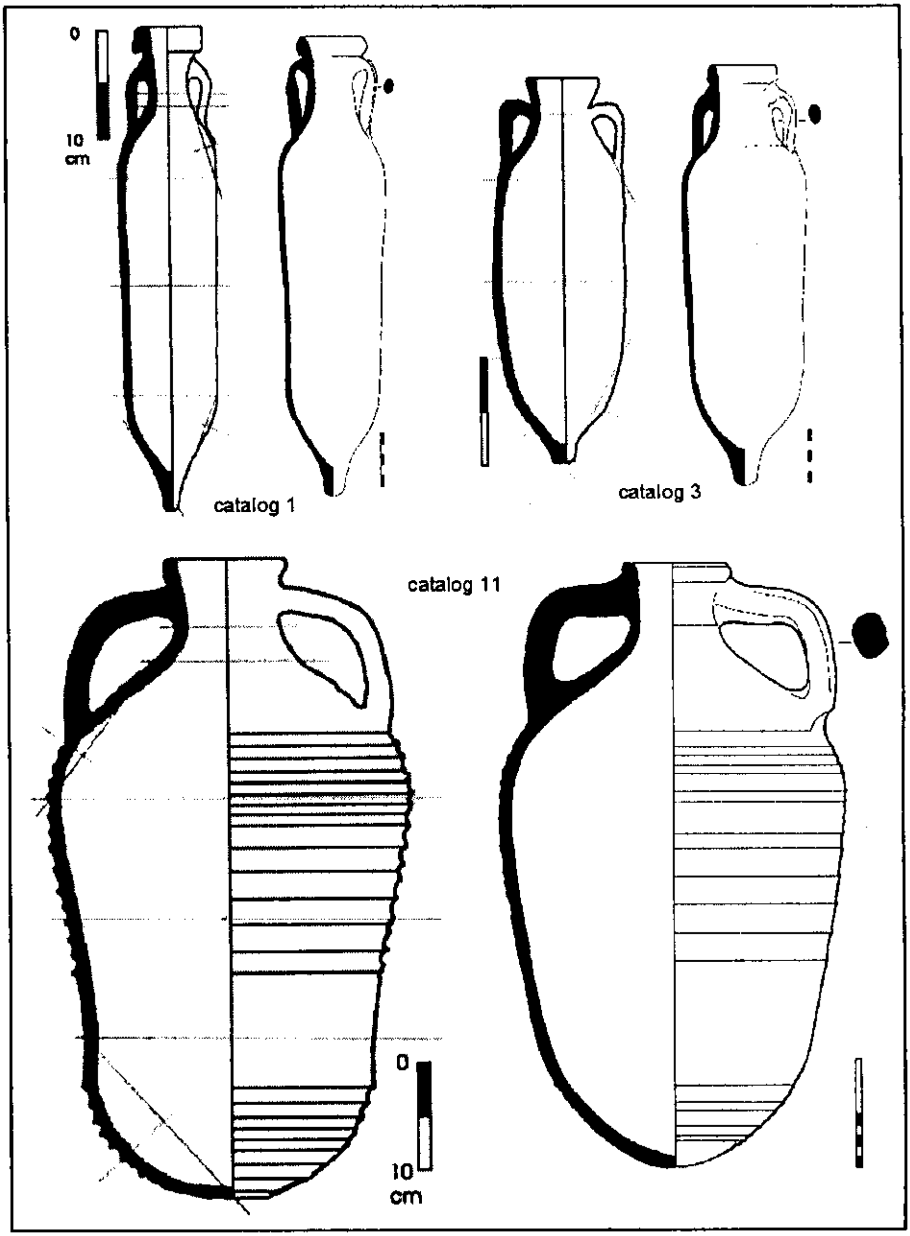


Fig. 1. – Pairs of illustrations from Opriş 2000 (left) and Opriş 2003 (right), representing the same artifacts: Capidava no. inv. 4488 (cat. 1), 5054 (cat. 3) and 4781 (cat. 11). Original graphics, excluding left marks from measurements in Compass System. Normalized scale 1:7.

Finally, I have had discover the simpler, but most astonishing fact, the kind that couldn't cross my mind: the drawings from 1999 were not those from 2003! Different is not only the graphic design, but the shape itself! I wander now only why Ioan C. Opriş was wandering why the figures in his table do not fit. How could they match?! Take a look on the figure 1! I have chosen there only the first three drawings corresponding to catalogue...

Let me be myself, the bad wolf, and put the things how they are: Ioan Opriş *new* that the drawings are bad, and changed them (doesn't matter here how). Having doubts about figures collected from bad drawings, he *measured* the capacity of the originals (as he can do: with water), and assembled together with Compass figures, on two columns; all flourished with exclamation marks and dressed in silence. For a professional – it's unlikely. For a *friend* – that's unspeakable.

THE SOLUTION

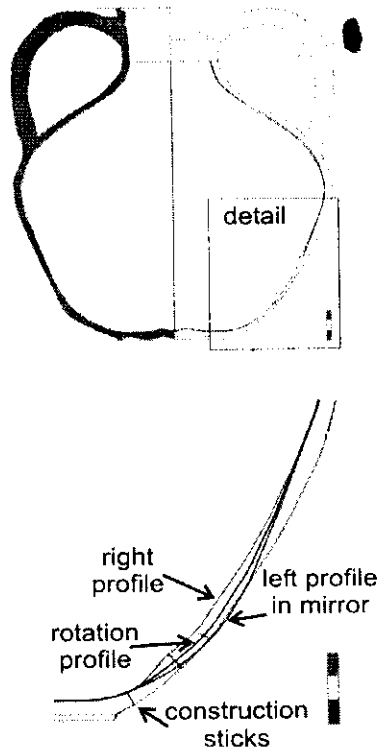
This section should be a little guidance for avoiding such embarrassing situations as that presented. At stake is not dr. Opriş, as someone could gloss, but Romanian archaeology. There are relatively recent pottery monographs, for the same Roman Scythia Minor, considered by many as paradigms of the modern archaeology, that not even mention the capacity issue, other way that in somebody else books (*Opaiţ 1996*; *Topoleanu 2000*). Ioan C. Opriş tried a step forward.

AutoCAD procedure is slow and hazardous. The operation is made with a rotated surface, i.e. one have to get the inner shape of the recipient, split by vertical (real) symmetry axis. First operation, and the most unpleasant, is to get a vector model for rotational surface, which could be accomplished in CorelDraw. Many traps are around: for instance, any line from Corel has a thickness, but would be preferable a line without any thickness; one can use the *wireframe view*, but the bitmap become blur. The line thickness is also dangerous working with scales (even using *transformation tool*, controlling therefore strictly the dimensions of the rectangle, half of chosen thickness *will be later measured*, enlarging the scale).

The most difficult step is to draw a surface for rotation that is representing fair *the both sides of the pottery shape* (Figure 2). The operation – described only through image, the text being too long – requires good skills and plenty of time. The step is not compulsive for most of amphorae, but those with symmetry problems could generate errors. For example, catalogue number 11 gave first time, with rotational surface taken only on one side, 20.711 l., and second time, with rotational surface made as a compromise for both sides, gave 19.859 l. (error 4.29%); the same for catalogue 175 (that from fig. 2) that gave 9.427 l. first time, and 9.026 l. second time (with the complete procedure; error 4.4%).

Other errors could be generated in ACAD itself. I will not describe the procedure, because would be useless for many readers. I should say anyway, for the

Fig. 2. – Corel procedure.



advanced PC users, that the position of the axis of symmetry (those asked by ACAD as reference for the surface) could be another source of error. Albeit a distanced axis produces a *torus*, the axis very close to surface does not, against our expectancy. And "very close" could mean one mm, or one micron (better last). Fortunately, ACAD aloud extreme zooming and we have to take advantage (else, the difference is error...). This is a certain possibility to get figures too high (around 0.5%) in ACAD.

For the Compass System there is a whole section, the last of this paper – a technical appendix – so I will make just an abridged exhibit of the general frame. Compass System is an Access Application, build up first (1996) for morphological reasons, and adapted later for capacity calculation. The main concept is an extremely simple one: to make an intricate thing easier by decomposing it simple components (like: the foot, the lower body, the middle section, the upper body, the neck). An older variant took the truncated con formula like a departure point, with corrections for vaulting. The later variant makes things even easier, considering just the truncated paraboloid formula (and the pendant – the truncated hyperboloid). This last produced the figures from this paper. One has to take note that it is working properly on normally vaulted body shape, but some (few) amphorae could look more like a truncated conoid. In the example from the Figure 3 (cat. 102) there are construction lines added (as in measurement procedure), as well as the conoid area, marked gray. In this particular case, the errors expected are low, because the conoid upper body is compensated by a conoid long neck, the form being, in a second analyze, a relatively regular shape. One has anyway to watch carefully which formula employs for which shape.

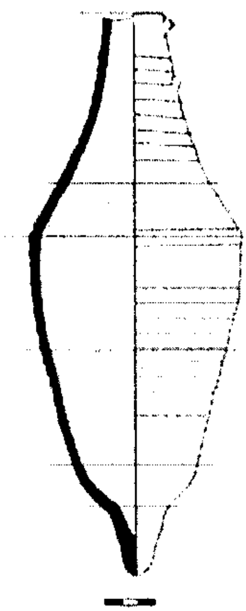


Fig. 3. – Conoid bodies.

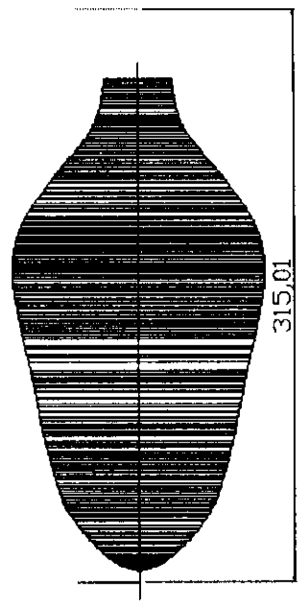


Fig. 4. – ACAD snapshot.

After this short excursus in computer assist methodology, I am coming back to archaeology to say that pouring something in a recipient to find out the capacity is far to be a new gadget, either claiming it as science (*Blake 1997*). The comparison between such *experiments* and mathematical calculations was already done, for the same geographical area and the same class of artifacts (*Conrad 1999*). It is quite interesting to put some facts face to face. Twenty-three amphorae from Iatrus have been measured by dividing the shape into "a large number of rectangles". Intuitively, Sven Conrad made exactly what AutoCAD do, but with hundreds of slices (fig. 4, cat. 128, shrunken image). The most part of the amphorae remained to mathematical calculation, being not complete. Nine of them gave also figures from a similar experiment from that of Capidava, by filling with water. The differences for recipients larger than 30 l. was in the gap 5–8%, but for the smaller ones it rose up to 20%. Dr. Conrad was concluded that in such conditions it's not possible to give an average[1].

[1] In the short time I had between the moment I decided to write the paper and the closing edition, I was not able to find Proceedings volume. I asked dr. Conrad informations about, that he

It looks that, for now, couldn't be done more. But I think that we have, today, all necessary tools for better solutions, closer to results with scientific relevance. An older solution is as simple as possible. We have to take shots with a telescopic objective, from relative distance, so that camera should be at least five times away compared with the artifact maximum dimension (for an amphora 50 cms. height – at least 2,5 m). I will not reproduce here the demonstration (*Teodor 1996*, 15; *Teodor 2001*, section 1.2.2), but in such condition the perspective deformation is below 1%, that is better than any hand draw (minimum error expected is around 4% for the best drawings; error determined following drawing experiment from *Orton & 1993*, p. 93, fig. 7.3). With digital cameras from our days, there is no problem to take a picture from ten times amphora's height, or more.

It is obvious to me that Ioan Opris couldn't do filling measurement errors greater than 5%. Not those figures are wrong, but the drawings! Finally – ironically – I know that he is... right, and all my computers, programs and skills can nothing without *a fair representation*. Lately, Dan Ştefan made me aware about photogrammetric techniques, allowing an engineering representation[2]. Basically, there are taken some photos from different angles (with the same focus), interpolated by machine and brought to known dimensions. The soft (Autodesk Land, for example) is excruciating expensive, but could be reached for free in a university frame. Of course, those ortho images could provide only the outer shape. For the inner shape we badly need the pottery wall thickness, and we should do *anything* to get it, including holes, with drilling machine, if we have to. In Capidava case, most of errors, I suspect, came exactly from the very good conservation state of the artifacts (complete or completed before drawing), and a casually presumption of the real thickness.

A discussion about a restrained lot of recipients couldn't go further with an analytic proof that amphorae capacities make sense, and more, what specific sense. A large lot, as that of 71 items, from Capidava, can be a good starting point to enlighten some about amphorae utility, because the statistic rule of averaging errors makes, at least for some frequent types, like Antonova V, or LR1, to have relatively clear classes of capacity. That would be, of course, another paper, if any.

provided it so kindly, by e-mail. I am thankfull for that! For other attempts to rationalize the volume issue, see *Hawthorne 1996*, *Senior & 1995*, *Wahlen 1998*.

[2] Dan Ştefan is student to Bucharest University, but the main contributor in bringing the ICT thechnologies into the service of Romanian archaeology. Personal communication. I am expecting the practical work too. See also *Lewis & 1991*.

APPENDIX

Calculation formula for capacities following Compass System

THE GENERAL CASE

The Compass System was borne as a morphological system for regular pottery (with round plan; see *Teodor 1996*; *Teodor 2001*, chapter 1). Later, it was adapted for capacity calculations (*Teodor 1998*; *2000*; *2001*, chapter 2). The main target was the barbarian pottery from Late Roman and Migrations, but the proposed goal was to enable analyses for any kind of recipients, no matter the culture. It was projected, also, to provide morphological and capacity data for either entire-shapes or fragments. This last detail is important for understanding the option for an analytical perspective, instead of fashionable shape description (vectors, splines, differential calculations; for example *Liming & 1989*; *Orton & 1993*, 152–163 for an overview).

We will pass over the morphological system, to expose the calculation for capacity. In *Figure 5* capitals are mark points, lower cases are diameters, and the numbers address the simple geometrical parts that make up the shape, named *sectors*. Sector 6 is not considered usable capacity and it is excluded from calculation.

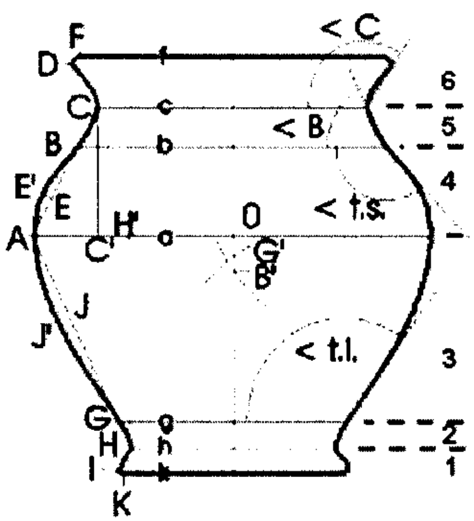


Fig. 5. – Measurements in Compass System.

List of arithmetical signs:

/ = divide
* = multiply
^ = involution
sqr = square root

There are used brackets programming like, pairs of opening-closing, no matter how many, with the same signification like in mathematics (priority in processing). There will be used also indentation (almost) program-ming like, to suggest the relationship, and sign ";" to detach formula of its explanation.

Each sector is considered a truncated con, with corrections for arching. Sectors 1 and 2 are considered together.

The section, not represented in *Figure 5*, is taken into consideration. All radii are without wall section. The upper section (noted *Mgrs*) is the average of measurements made in several points across upper body (mainly in marks B and E'). The lower section (noted *Mgri*) is an average for mark K and the middle of the bottom, on the symmetry axis.

All factors beginning with letter M are measurement values. *Mscale* is the scale of the measured drawing (only the denominator, like "3" from 1:3). For others – see *Figure 5*. Factors beginning with U are angles (from the mark *X*). Factors beginning with R are radius (without wall section). Factors are closed in rectangular brackets.

Notations for heights:

MI = total height (sectors 1 to 6)
MIs = upper height (sectors 4 to 6)
lower height: not measured (deduced as MI – MIs)
Mhk = sector 1
Mgk = sectors 1+2
Mbf = sectors 5-6
Mbc = sector 5
sectors 3 and 4: not measured (deduced)

Other factors:

Mc, Ma, Mh, Mk = diameters in *c, a, h, k* (see *Figure 5*)

All sectors are calculated forwarding the truncated con formula for capacity (or volume):

$$V = Pi/3 * h (R^2 + r^2 + rR)$$ (see *Memorator 1958*, p. 171)

where V = volume; h = height; R = major radius; r = minor radius; Pi = ∏ (3.141592653);

Sector 1+2

```
(1,0467*                                                        ; Pi/3
(([Mgk]-[Mgri]-[Mgd])*[Mscale])*                                ; h
((((((([Mh]/2)-[Mgri])*[Mscale])^2)+                            ; r^2
((([GO]-[Mgrs])*[Mscale])^2))+                                  ; R^2
((([GO]-[Mgrs])*[Mscale])*((([Mh]/2)-[Mgri])*[Mscale]))))       ; rR
*0,000001          ; transformation from microliters to liters (measurements in mms.)
                   ; note: no correction applied;
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Sector 3

```
(1,0467*                                                        ; Pi/3
(([MI]-[MIs]-[Mgk])*[Mscale])*                                  ; h
(([R_A]^2)+([R_G]^2)+([R_G]*[R_A]))                             ; R^2 + r^2 + rR
*0,000001)+[SS_3]               ; transformation + correction for vaulting,
```

where:

R_A = (([Ma]/2)-[Mgrs])*[Mscale]
R_G = ([GO]-[Mgrs])*[Mscale] where
GO = Sqr((([Ma]/2)^2)-((([MI]-[MIs]-[Mgk])+[MOG'])^2))

; trigonometric deduction for radius in G (diameter in G not taken by measurement), like $c^2 = a^2 - b^2$;

; MOG' is the measurement from *O* (intersection between diameter *a* and symmetry axis *o*) to *G'* (see *Fig. 5*); *G'* is the intersection of the circular arc drawn from mark *G*, with a radius equal with radius of the diameter *a*, at the intersection with the symmetry axis; values upper *O* are positive; values under *O* are negative;

SS_3 = (2,094395102* ; $Pi / 3 * 2$
(([R_A]+(([MI]-[MIs]-[Mgk])*[Mscale])/2)^2)* ; R^2
([MJJ']*[Mscale])* ; h
0,000001)* ; transformation to litres
(1-(([MAJ]/[MAG])/10)) ; correction for "the place of the maximum vaulting" (MAJ/MAG, see *Fig. 5*); note that the value of the fraction is in inverse ratio with the amplitude of the correction;

Sector 4

(1,0467* ; $Pi/ 3$
(([MIs]-[Mbf])*[Mscale])* ; h
(([R_A]^2)+([R_B]^2)+([R_B]*[R_A]))* ; $R^2 + r^2 + rR$
0,000001)+[SS_D] ; transformation + correction for vaulting,
where:
R_A = (([Ma]/2)-[Mgrs])*[Mscale]
R_B = ([BO]-[Mgrs])*[Mscale]; where
BO: Sqr((([Ma]/2)^2)-((([MIs]-[Mbf])-[MOB'])^2))
; trigonometric deduction for radius in B (diameter in B not taken by measurement), like $c^2 = a^2 - b^2$;
; MOB' is the measurement from *O* (intersection between diameter *a* and symmetry axis *o*) to *B'* (see *Fig. 5*); *G'* is the intersection of the circular arc drawn from mark *B*, with a radius equal with radius of the diameter *a*, at the intersection with the symmetry axis; values upper *O* are positive; values under *O* are negative;

Sector 5

(1,0467* ; $Pi/ 3$
([Mbc]*[Mscale])* ; h
((((([Mc]/2)-[Mgrs])*[Mscale])^2)+ ; r^2
((([BO]-[Mgrs])*[Mscale])^2)+ ; R^2
((([BO]-[Mgrs])*[Mscale])*((([Mc]/2)-[Mgrs])*[Mscale])))* ; Rr
0,000001)* ; transformation
(1-((([Uts]-90)-(90-[UB]))/100)); correction for vaulting,
where:
BO = Sqr((([Ma]/2)^2)-((([MIs]-[Mbf])-[MOB'])^2)); see *Sector 4*

note that the correction is negative; the amplitude of the correction is given by the difference between the angle *ts* and the angle from *B*; higher the difference, greater the correction (normal values between 0.81 and 0.95);

note: a similar correction could be theoretically applied to sector 2, but Compass System does not consider the angle from G (as angle from B in sector 5); in fact, the shapes with well developed foot (similar to the neck) are quite rare, except some prehistoric ones; for the last, the angle from G should be considered, as well for morphology as for capacity reasons;

Note: sectors 1–5 are summed; figures for half-shapes (either upper or lower half) are produced independently.

AMPHORA CASE

Amphora is a special case, as well as a morphological issue and as a capacity one. First of all, it comes a seventh *sector*, between third and fourth, sector that is quasi-cylindrical or quasi-conical. As we will further see, it is always considered a cylinder (as starting formula). The new diameters appeared, *MaS* (the superior "middle" diameter) and *MaI* (the inferior "middle" diameter) are at the edge of the quasi-cylindrical sector, *Ma* being on the half height of the sector. The average between the three diameters (MaS, Ma, MaI) makes cylinder's diameter, the vaulting lost being feeble.

The second specific is that the amphora segments are almost anytime very close to a truncated paraboloid (or hyperboloid), that aloud us to use a simpler formula, as implementation, without any correction necessary:

truncated paraboloid volume: $1/2\ Pi\ (R^2 + r^2)\ h$

(see *Memorator 1958*, p. 175)

I didn't find in my books formula for the *truncated hyperboloid*, but I tried to find it out from the relationship between the paraboloid and conoid truncs. It seems very much to be: $Pi * hrR$. This presumption was verified with the relation *trunc. con – (parab. trunc – trunc. con)*, and the results are pretty tight (the errors seem to be the consequences of the imperfect number *Pi*).

A simpler formula means also fewer measurements: 6 for establishing so-called "format" (MOB' and MOG', above; see also Fig. 5)) and 12 for arch measurements. The job will be ready faster, with almost the same result. For a change, we need now a new method for determining the diameter from G (both "format" and vaulting measurements being lost). We could just measure it, but should be no "added value" for the morphological description. I decided then to pick the angle from G (see *Figure 6*), which affords the trigonometric deduction.

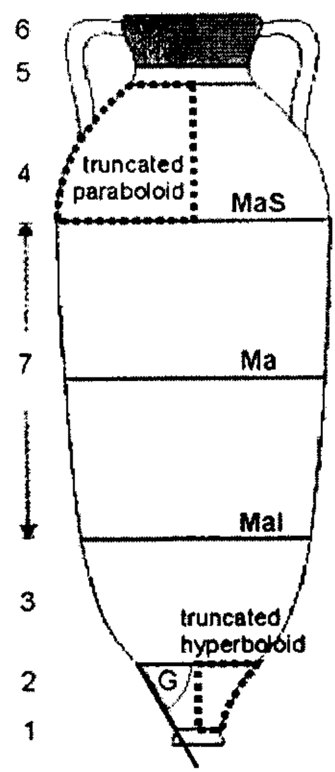

Fig. 6.

Sector 1-2 (truncated con)

```
1,0467*                                          ; Pi/ 3
(([Mgk]-[Mgri]-[Mgd])*[Mscale])*                 ; h
(([R_G]^2)+([R_K]^2)+([R_G]*[R_K]))*             ; R^2 + r^2 + rR
0,000001                  ; transformation
```

where

```
Mgd = height for the concave bottom (if any; by default 0)
R_G = (([Diam_G]/2)-[Mgrs])*[Mscale]
R_K = (([Mh]/2)-(([Mgrs]+[Mgri])/2))*[Mscale]
```

note that if the external shape of the foot is a hyperboloid, the inner shape is not; the correction for the wall section is the only one necessary; usually, the values for sector 1–2 are very low, or even negative

Sector 3 (truncated paraboloid)

```
1,570796*                                        ; Pi/ 2
(([MI]-[MIs]-[Mîmd]-[Mgk])*[Mscale])*            ; h
(([R_Ai]^2)+([R_G]^2))*                          ; R^2 + r^2
0,000001                  ; transformation
```

where

```
Midm =     height of the sector 7
R_Ai =     (([MaI]/2)-[Mgrs])*[Mscale]
R_G =      (([Diam_G]/2)-[Mgrs])*[Mscale]and
Diam_G =   ((([Mgk]-[Mhk])*                 ; sector's 2 height (= cathetus)
((Cos([UG_R]))/(Sin([UG_R]))))              ; cotangent of the opposed angle
*2)                                         ; all taken twice and...
+[Mh]                                       ; added to the diameter in H (see Fig. 5)
```

where

```
UG_R = [UG]*0,0174533                       ; radians for angle from G
```

Sector 7 (cylinder, respectively $Pi * R^2 h$)

```
3,141592653*                                ; Pi
((([R_As]+[R_A]+[R_Ai])/3)^2)*              ; R^2
([MImd]*[Mscale])*                          ; h
0,000001                                    ; transformation
```

Sector 4 (truncated paraboloid)

```
1,570796*; Pi/ 2
(([MIs]-[Mbf])*[Mscale])*                   ; h
(([R_As]^2)+([R_B]^2))*                     ; R^2 + r^2
0,000001                                    ; transformation
```

where

```
R_Ai = (([MaI]/2)-[Mgrs])*[Mscale]
R_B = (([Diam_B]/2)-[Mgrs])*[Mscale]        and
Diam_B = (([Mbc]*                           ; sector's 5 height (= cathetus)
((Cos([UB_R]))/(Sin([UB_R]))))              ; cotangent of the opposed angle
*2)                                         ; all taken twice and...
+[Mc]                                       ; added to the diameter in C (see Fig. 5)
```

where

UB_R = [UB]*0,0174533 ; radians for angle from B

Sector 5 (truncated hyperboloid)

3,141592653* ; *Pi*
([Mbc]*[Mscale])* ; *h*
[R_B]*[R_C]* ; *rR*
0,000001 ; transformation

where

R_B ; radius in *B*, see above
R_C = (([Mc]/2)-[Mgrs])*[Mscale] ; radius in *C*

Of course, strictly mathematical, all transformations could be made once, along the sum: (Sector_1+2 + Sector_3 + Sector_7 + Sector_4 + Sector_5) * 0,000001. As I already mentioned, Compass System works with partial capacities too. All sectors are displayed on the screen, as "passive" controls; they are not "usefull", being neither "input" (measurement), nor "output" (final result), but their presence on the screen can prevent errors on input (when passive controls look aberrant). The diameters from *B* and *G* are also "passive" controls, because those dimensions are not measured, but calculated. So, what could look just like fancy maths, turns into necessary check-points, that proves that measurements and recordings are all right, or at least, are not simply mad...

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